Math 218D: Week 5 Discussion

Study Copy

September 23, 2021

Problem 1. To verify if $\boldsymbol{v} \in \text{Null}(A)$ we must check the equation ____________________.

Problem 2. Show that $\boldsymbol{v} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ is in the null space of $A = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 1 & 3 \\ 2 & 1 & 4 \\ 0 & 1 & 2 \end{bmatrix}$.

Problem 3. Suppose A is 5×9 and that $\boldsymbol{v} \in \text{Null}(A)$. Then $\boldsymbol{v} \in \mathbb{R}^{\_\_\_}$.

Problem 4. Suppose that A has four columns related by the equation $\text{Col}_4 = 3\ \text{Col}_1 + \text{Col}_2 - \text{Col}_3$. Find a nonzero vector $\boldsymbol{v} \in \text{Null}(A)$.

Problem 5. A scalar λ is an *eigenvalue* of A if ____________________________.

Problem 6. The *eigenspace* of A corresponding to an eigenvalue λ is $\mathcal{E}_A(\lambda) =$ ____________________.

Problem 7. An *eigenvector* $\boldsymbol{v} \in \mathcal{E}_A(\lambda)$ satisfies the equation ____________________.

Problem 8. Show that $\boldsymbol{v} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ is an eigenvector of $A = \begin{bmatrix} 10 & -6 & 6 \\ 6 & -2 & 6 \\ -6 & 6 & -2 \end{bmatrix}$ and identify the eigenvalue.

Problem 9. Find all vectors in $\mathcal{E}_A(-3)$ where $A = \begin{bmatrix} -23 & 40 & -60 \\ -5 & 7 & -15 \\ 5 & -10 & 12 \end{bmatrix}$.

Problem 10. To verify if $\boldsymbol{v} \in \operatorname{Col}(A)$ we must check the equation ____________________.

Problem 11. Determine if $\boldsymbol{v} = \begin{bmatrix} 6 \\ 12 \\ 2 \end{bmatrix}$ is in the column space of $A = \begin{bmatrix} 1 & -3 & 4 \\ 2 & -6 & 8 \\ -3 & 9 & -12 \end{bmatrix}$.

Problem 12. To verify if $\boldsymbol{v} \in \operatorname{Span}\{\boldsymbol{v}_1, \dots, \boldsymbol{v}_k\}$ we must check ____.

Problem 13. Determine if $\begin{bmatrix} 3 & 0 & 1 \end{bmatrix}^\intercal \in \operatorname{Span}\{\begin{bmatrix} 1 & -3 & -3 \end{bmatrix}^\intercal, \begin{bmatrix} -1 & 4 & 3 \end{bmatrix}^\intercal\}$.

Math 218D: Week 6 Discussion

Study Copy

September 30, 2021

Problem 1. By definition, what does it mean to call a list of vectors $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_n\}$ *linearly dependent*?

Problem 2. By definition, what does it mean to call a list of vectors $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_n\}$ *linearly independent*?

Problem 3. Determine if $\{\begin{bmatrix}1 & -3 & 1\end{bmatrix}^\intercal, \begin{bmatrix}-4 & 13 & -3\end{bmatrix}^\intercal, \begin{bmatrix}5 & -17 & 3\end{bmatrix}^\intercal\}$ is independent.

Problem 4. Suppose that $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3 \in \mathbb{R}^n$ and let A be an $m \times n$ matrix such that $\{A\boldsymbol{v}_1, A\boldsymbol{v}_2, A\boldsymbol{v}_3\}$ is linearly independent. Show that $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ is linearly independent.

Problem 5. The columns of a matrix A are independent if and only if ____________________________.

Problem 6. Consider the calculations

$$\operatorname{rref}\overbrace{\begin{bmatrix} 9 & 4 & 4 \\ -36 & -16 & -16 \\ 20 & 9 & 4 \\ -49 & -22 & -12 \end{bmatrix}}^{A} = \begin{bmatrix} 1 & 0 & 20 \\ 0 & 1 & -44 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \operatorname{rref}\overbrace{\begin{bmatrix} 9 & -36 & 20 & -49 \\ 4 & -16 & 9 & -22 \\ 4 & -16 & 4 & -12 \end{bmatrix}}^{A^\intercal} = \begin{bmatrix} 1 & -4 & 0 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find the pivot solutions to $A\boldsymbol{v} = \boldsymbol{O}$. These vectors form a basis of ______________.

(b) Find the pivot solutions to $A^\intercal\boldsymbol{v} = \boldsymbol{O}$. These vectors form a basis of ______________.

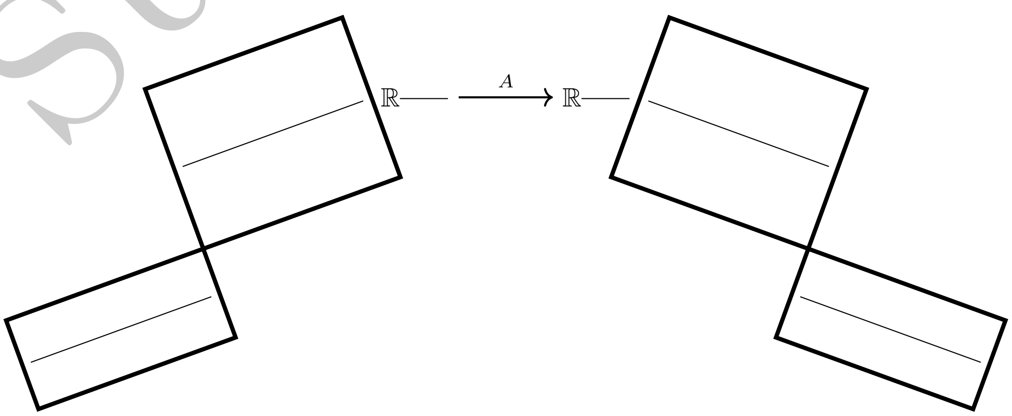
(c) Find the pivot columns of A. These vectors form a basis of ______________.

(d) Find the nonzero rows of rref(A). These vectors form a basis of ______________.

(e) The pivot columns of $A^\intercal$ form a basis of ______________.

(f) The nonzero rows of rref($A^\intercal$) form a basis of ______________.

(g) Fill in the blanks in the figure below.



Math 218D: Week 7 Discussion

Study Copy

October 7, 2021

Problem 1. Suppose that A is a matrix satisfying

$$\operatorname{Col}(A^\intercal) = \operatorname{Span}\{\begin{bmatrix}1 & 1 & 0 & 1\end{bmatrix}^\intercal, \begin{bmatrix}0 & 3 & 2 & 4\end{bmatrix}^\intercal\} \qquad \operatorname{Null}(A^\intercal) = \operatorname{Span}\{\begin{bmatrix}2 & 1 & 1\end{bmatrix}^\intercal\}$$

(a) Draw the picture of the four fundamental subspaces of A, including their dimensions

(b) Determine if $\boldsymbol{v} = \begin{bmatrix}1 & 0 & 2 & -1\end{bmatrix}^\intercal$ satisfies $A\boldsymbol{v} = \boldsymbol{O}$.

(c) Determine if $\boldsymbol{b} = \begin{bmatrix}3 & 5 & 2\end{bmatrix}^\intercal$ makes the system $A\boldsymbol{x} = \boldsymbol{b}$ consistent.

(d) Explain why $\operatorname{Null}(A) \neq \operatorname{Span}\{\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}^\intercal, \begin{bmatrix}2 & 1 & 0 & 0\end{bmatrix}^\intercal\}$.

Problem 2. Suppose $EA = R$ where

$$E = \begin{bmatrix} 1 & -3 & -1 & 17 \\ -3 & 10 & 5 & -56 \\ 5 & -19 & -12 & 105 \\ -1 & 7 & 7 & -36 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & -3 & 0 & -7 & 0 \\ 0 & 0 & 1 & 9 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Draw the picture of the four fundamental subspaces of A, including their dimensions

(b) Find a basis of $\text{Col}(A^\mathsf{T})$.

(c) Find a basis of $\text{Null}(A^\mathsf{T})$.

(d) Find a basis of $\text{Null}(A)$.

(e) Find a basis of $\text{Col}(A)$.

Math 218D: Week 8 Discussion

Study Copy

October 14, 2021

Problem 1. The *least squares problem* associated to $A\boldsymbol{x} = \boldsymbol{b}$ is ____________________.

Problem 2. Suppose $\widehat{\boldsymbol{x}}$ is a least squares approximate solution to $A\boldsymbol{x} = \boldsymbol{b}$. Then $A\widehat{\boldsymbol{x}} =$ ____________________.

Problem 3. The *least squares error* is defined as ____________________.

Problem 4. Define the concept of an $A = QR$ *factorization*.

Problem 5. A matrix M has orthonormal columns if and only if $M^{\intercal}M =$ __________.

Problem 6. Given $A = QR$, projection onto $\operatorname{Col}(A)$ is given by $P_{\operatorname{Col}(A)} =$ __________.

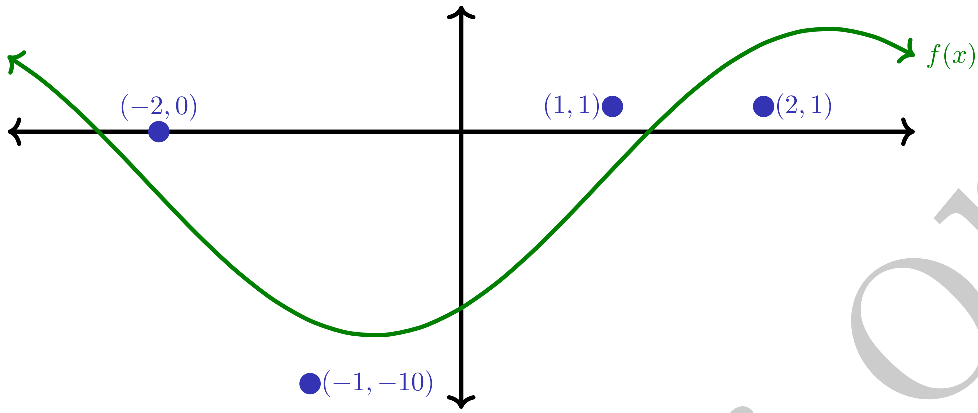
Problem 7. Suppose $A = QR$ where A has full column rank. Then the least squares problem $A^{\intercal}A\widehat{\boldsymbol{x}} = A^{\intercal}\boldsymbol{b}$ reduces to ____________________.

Problem 8. Suppose that A is $m \times n$ with orthonormal columns and that $\boldsymbol{v} \in \mathbb{R}^n$.

(*a*) Show that $\|A\boldsymbol{v}\| = \|\boldsymbol{v}\|$.

(*b*) Show that $n \leq m$.

Problem 9. The figure below depicts the result of using the technique of least squares to fit a curve of the form $f(x) = c_0 + c_1 \cos(\pi x/3) + c_2 \sin(\pi x/3)$ to four data points.



Find the values of c_0, c_1, and c_2 and calculate the error in using $f(x)$ to approximate this data.