Math 218D: Week 5 Discussion

Study Copy

September 29, 2022

Problem 1. To verify if $\boldsymbol{v} \in \operatorname{Null}(A)$ we must check the equation ____________________.

Problem 2. Show that $\boldsymbol{v} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ is in the null space of $A = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 1 & 3 \\ 2 & 1 & 4 \\ 0 & 1 & 2 \end{bmatrix}$.

Problem 3. Suppose A is 5×9 and that $\boldsymbol{v} \in \operatorname{Null}(A)$. Then $\boldsymbol{v} \in \mathbb{R}^{\_\_\_}$.

Problem 4. Suppose that A has four columns related by the equation $\operatorname{Col}_4 = 3\operatorname{Col}_1 + \operatorname{Col}_2 - \operatorname{Col}_3$. Find a nonzero vector $\boldsymbol{v} \in \operatorname{Null}(A)$.

Problem 5. A scalar λ is an *eigenvalue* of A if ______________________________.

Problem 6. The *eigenspace* of A corresponding to an eigenvalue λ is $\mathcal{E}_A(\lambda) =$ ______________________.

Problem 7. An *eigenvector* $\boldsymbol{v} \in \mathcal{E}_A(\lambda)$ satisfies the equation ______________________.

Problem 8. Show that $\boldsymbol{v} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ is an eigenvector of $A = \begin{bmatrix} 10 & -6 & 6 \\ 6 & -2 & 6 \\ -6 & 6 & -2 \end{bmatrix}$ and identify the eigenvalue.

Problem 9. Find all vectors in $\mathcal{E}_A(-3)$ where $A = \begin{bmatrix} -23 & 40 & -60 \\ -5 & 7 & -15 \\ 5 & -10 & 12 \end{bmatrix}$.

Problem 10. To verify if $\boldsymbol{v} \in \operatorname{Col}(A)$ we must check the equation ________________________.

Problem 11. Determine if $\boldsymbol{v} = \begin{bmatrix} 6 \\ 12 \\ 2 \end{bmatrix}$ is in the column space of $A = \begin{bmatrix} 1 & -3 & 4 \\ 2 & -6 & 8 \\ -3 & 9 & -12 \end{bmatrix}$.

Problem 12. To verify if $\boldsymbol{v} \in \operatorname{Span}\{\boldsymbol{v}_1, \dots, \boldsymbol{v}_k\}$ we must check ____.

Problem 13. Determine if $\begin{bmatrix} 3 & 0 & 1 \end{bmatrix}^\intercal \in \operatorname{Span}\{\begin{bmatrix} 1 & -3 & -3 \end{bmatrix}^\intercal, \begin{bmatrix} -1 & 4 & 3 \end{bmatrix}^\intercal\}$.

Math 218D: Week 6 Discussion

Study Copy

October 6, 2022

Problem 1. By definition, what does it mean to call a list of vectors $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_n\}$ *linearly dependent*?

Problem 2. By definition, what does it mean to call a list of vectors $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_n\}$ *linearly independent*?

Problem 3. Determine if $\{\begin{bmatrix}1 & -3 & 1\end{bmatrix}^\intercal, \begin{bmatrix}-4 & 13 & -3\end{bmatrix}^\intercal, \begin{bmatrix}5 & -17 & 3\end{bmatrix}^\intercal\}$ is independent.

Problem 4. Suppose that $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3 \in \mathbb{R}^n$ and let A be an $m \times n$ matrix such that $\{A\boldsymbol{v}_1, A\boldsymbol{v}_2, A\boldsymbol{v}_3\}$ is linearly independent. Show that $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ is linearly independent.

Problem 5. The columns of a matrix A are independent if and only if ____________________________.

Problem 6. Suppose that A is a 3×3 matrix satisfying the following three equations.

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}1\\-2\\-1\end{bmatrix} \qquad \operatorname{rref}\left[\begin{array}{ccc|c} & & & 1\\ & A & & 1\\ & & & 1\end{array}\right] = \left[\begin{array}{ccc|c}1 & 0 & 1 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\end{array}\right] \qquad \operatorname{rref}\left[\begin{array}{ccc|c} & & & 1\\ & A^\intercal & & 1\\ & & & 1\end{array}\right] = \left[\begin{array}{ccc|c}1 & 0 & 1 & -11\\0 & 1 & 1 & -7\\0 & 0 & 0 & 0\end{array}\right]$$

Note that $\operatorname{rref}(A)$ and $\operatorname{rref}(A^\intercal)$ can be inferred from the second and third equations above.

(a) The vector $\begin{bmatrix}1 & 1 & 1\end{bmatrix}^\intercal$ belongs to exactly one of the four fundamental subspaces of A. Select this space.

○ The null space. ○ The row space. ○ The column space. ○ The left null space.

(b) The vector $\begin{bmatrix}1 & -2 & -1\end{bmatrix}^\intercal$ belongs to exactly one of the four fundamental subspaces of A. Select this space.

○ The null space. ○ The row space. ○ The column space. ○ The left null space.

(c) Determine if $\begin{bmatrix}1 & -1 & 1\end{bmatrix}^\intercal \in \operatorname{Null}(A)$.

Problem 7. Suppose that A and B are $n \times n$ matrices and that $\boldsymbol{v} \in \mathbb{R}^n$ satisfies $\boldsymbol{v} \in \mathcal{E}_A(-2)$ and $\boldsymbol{v} \in \mathcal{E}_B(5)$. Show that $\boldsymbol{v}$ is an eigenvector of $M = A^2 + AB - I_n$ and identify the corresponding eigenvalue.

Math 218D: Week 7 Discussion

Study Copy

October 13, 2022

Problem 1. Consider the calculations

$$\operatorname{rref}\overset{A}{\begin{bmatrix} 9 & 4 & 4 \\ -36 & -16 & -16 \\ 20 & 9 & 4 \\ -49 & -22 & -12 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 20 \\ 0 & 1 & -44 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \operatorname{rref}\overset{A^{\intercal}}{\begin{bmatrix} 9 & -36 & 20 & -49 \\ 4 & -16 & 9 & -22 \\ 4 & -16 & 4 & -12 \end{bmatrix}} = \begin{bmatrix} 1 & -4 & 0 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find the pivot solutions to $A\boldsymbol{v} = \boldsymbol{O}$. These vectors form a basis of ______________.

(b) Find the pivot solutions to $A^{\intercal}\boldsymbol{v} = \boldsymbol{O}$. These vectors form a basis of ______________.

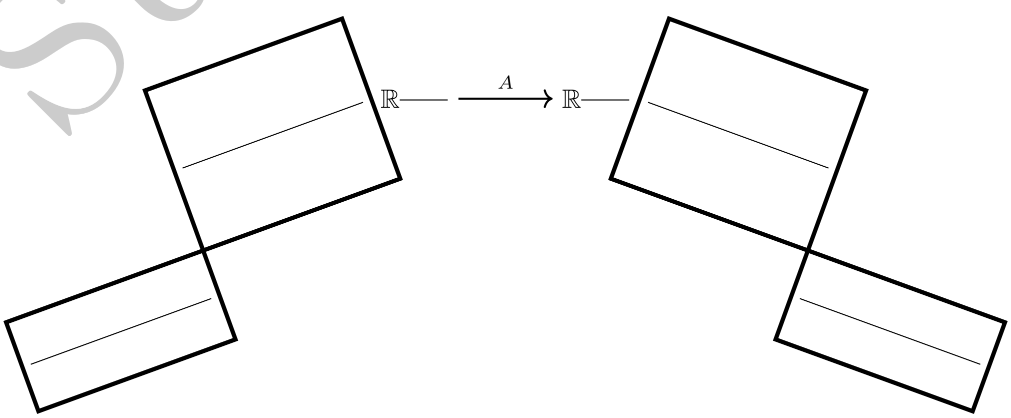
(c) Find the pivot columns of A. These vectors form a basis of ______________.

(d) Find the nonzero rows of $\operatorname{rref}(A)$. These vectors form a basis of ______________.

(e) The pivot columns of $A^{\intercal}$ form a basis of ______________.

(f) The nonzero rows of $\operatorname{rref}(A^{\intercal})$ form a basis of ______________.

(g) Fill in the blanks in the figure below.



Problem 2. Suppose $EA = R$ where

$$E = \begin{bmatrix} 1 & -3 & -1 & 17 \\ -3 & 10 & 5 & -56 \\ 5 & -19 & -12 & 105 \\ -1 & 7 & 7 & -36 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & -3 & 0 & -7 & 0 \\ 0 & 0 & 1 & 9 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Draw the picture of the four fundamental subspaces of A, including their dimensions

(b) Find a basis of $\mathrm{Col}(A^{\intercal})$.

(c) Find a basis of $\mathrm{Null}(A^{\intercal})$.

(d) Find a basis of $\mathrm{Null}(A)$.

(e) Find a basis of $\mathrm{Col}(A)$.

Math 218D: Week 8 Discussion

Study Copy

October 20, 2022

Problem 1. Suppose that A is a matrix satisfying

$$\mathrm{Col}(A^\intercal) = \mathrm{Span}\{\begin{bmatrix}1 & 1 & 0 & 1\end{bmatrix}^\intercal, \begin{bmatrix}0 & 3 & 2 & 4\end{bmatrix}^\intercal\} \qquad \mathrm{Null}(A^\intercal) = \mathrm{Span}\{\begin{bmatrix}2 & 1 & 1\end{bmatrix}^\intercal\}$$

(a) Draw the picture of the four fundamental subspaces of A, including their dimensions

(b) Determine if $\boldsymbol{v} = \begin{bmatrix}1 & 0 & 2 & -1\end{bmatrix}^\intercal$ satisfies $A\boldsymbol{v} = \boldsymbol{O}$.

(c) Determine if $\boldsymbol{b} = \begin{bmatrix}3 & 5 & 2\end{bmatrix}^\intercal$ makes the system $A\boldsymbol{x} = \boldsymbol{b}$ consistent.

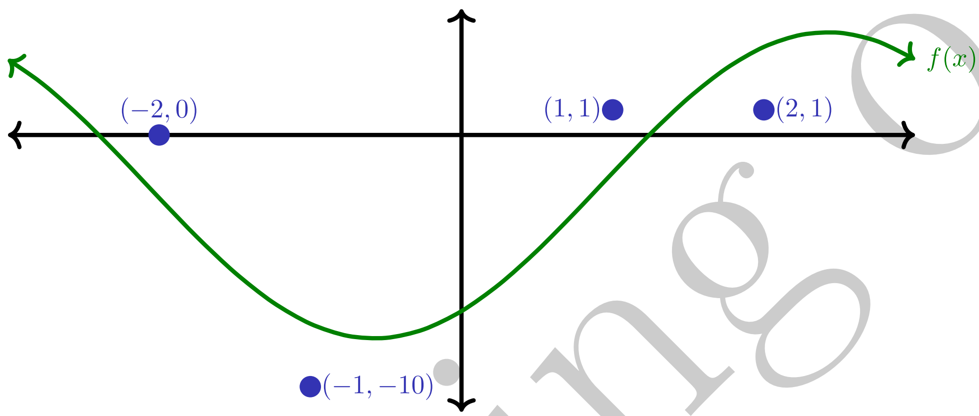
(d) Explain why $\mathrm{Null}(A) \neq \mathrm{Span}\{\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}^\intercal, \begin{bmatrix}2 & 1 & 0 & 0\end{bmatrix}^\intercal\}$.

Problem 2. The *least squares problem* associated to $A\boldsymbol{x} = \boldsymbol{b}$ is ____________________________.

Problem 3. Suppose $\widehat{\boldsymbol{x}}$ is a least squares approximate solution to $A\boldsymbol{x} = \boldsymbol{b}$. Then $A\widehat{\boldsymbol{x}} =$ ____________________________.

Problem 4. The *least squares error* is defined as ____________________________.

Problem 5. The figure below depicts the result of using the technique of least squares to fit a curve of the form $f(x) = c_0 + c_1 \cos(\pi x/3) + c_2 \sin(\pi x/3)$ to four data points.



Find the values of c_0, c_1, and c_2 and calculate the error in using $f(x)$ to approximate this data.