

Cholesky Factorizations

Problem 1. Find the definiteness of $S = \begin{bmatrix} 4 & -2 & -2 & -20 \\ -2 & 1 & 1 & 10 \\ -2 & 1 & 17 & 14 \\ -20 & 10 & 14 & 105 \end{bmatrix}$.

Problem 2. Calculate $S = LDL^T$ where $S = \begin{bmatrix} 2 & 12 & 10 \\ 12 & 67 & 105 \\ 10 & 105 & -351 \end{bmatrix}$.

Problem 3. Suppose that factoring $S = LDL^T$ allows us to write the quadratic form $q(\mathbf{x}) = \langle \mathbf{x}, S\mathbf{x} \rangle$ as

$$q(\mathbf{x}) = 10(x_1 - 5x_2 + 2x_3)^2 - 11(x_2 - 6x_3)^2 - 5x_3^2$$

Find L and D and determine the definiteness of S .

Problem 4. Determine the definiteness of $S = \begin{bmatrix} 0 & 4 & -6 & 8 & 16 \\ 4 & -650 & 3 & 1941 & -1 \\ -6 & 3 & 8 & -144 & 16 \\ 8 & 1941 & -144 & 2 & 18 \\ 16 & -1 & 16 & 18 & 52 \end{bmatrix}$.

Problem 5. Find R the Cholesky factorization $S = R^T R$ of $S = \begin{bmatrix} 9 & 15 & -6 \\ 15 & 29 & 8 \\ -6 & 8 & 206 \end{bmatrix}$.