

The Hessian

Problem 1. Consider the point $P(17, -22, 37)$ and suppose $f \in \mathcal{C}(\mathbb{R}^3)$ is a scalar field satisfying

$$Hf(P) = \begin{bmatrix} -4 & 2 & -2 \\ 2 & -2 & 3 \\ -2 & 3 & -8 \end{bmatrix}$$

Suppose we use the local linearization of f at P to estimate $f(17 - 1, -22 + 1, 37 + 1)$. Do we expect this estimation to be an overestimate or an underestimate? Explain.

Problem 2. Consider the point $P(1, 1, 1)$ and the scalar field $f \in \mathcal{C}(\mathbb{R}^3)$ given by

$$f(x, y, z) = x^3z + y^3 + x^2z + z^3$$

(a) Use the local linearization of f at P to approximate $f(1 + 1/4, 1 - 1/3, 1 - 1/5)$.

(b) Use the second degree Taylor polynomial of f at P to approximate $f(1 + 1/4, 1 - 1/3, 1 - 1/5)$.

(c) Do you expect your linear approximation in part (a) to be an overestimate or underestimate?